Calculating buckling capacity of built-up beams and columns

By Donald A. Bender, PE;
Robert E. Kimble, PE; Frank E. Woeste, PE

Abstract

Built-up beams and columns made of dimension lumber fastened with nails, screws or bolts are common in post-frame buildings. The current design practice for calculating buckling capacity is to use the lower-tail modulus of elasticity (E_{min}) value tabulated for dimension lumber; however, this ignores the averaging effect on E_{min} that occurs when the mechanically fastened laminations are constrained to deflect equally. In this paper, we describe a calculation method developed by Kimble and Bender (2010) that proposes a factor (C_s) to adjust the E_{min} value used in column and beam buckling calculations to account for the reduced variability of E when the laminations are constrained to deflect together. Using C_s to account for the reduction of E_{min} variability increases buckling capacity and produces more efficient designs for built-up beams and columns, provided they are designed, detailed, and installed in a manner that constrains the individual members to a common deflected shape. Design examples are provided for a nail-laminated column and multi-ply header utilizing the averaging effect on E_{min} that occurs when the mechanically fastened laminations are constrained to deflect in equal amounts.

Introduction

Built-up beams and columns from dimension lumber fastened with nails, screws or bolts are common in post-frame construction. These types of built-up members, as compared to adhesively bonded members such as glued-laminated timbers, do not achieve complete composite action due to the slip that must occur before the fasteners begin to transfer load. The degree of composite action depends on fastener type, size and location and can be further complicated by other effects such as dimensional changes due to moisture cycling and variations in lumber specific gravity and modulus of elasticity (E).

The National Design Specification for Wood Construction (American Forest and Paper Association, 2005) accounts for load-sharing "system effects" of built-up assemblies through adjustment factors such as the C_r repetitive member factor for dimension lumber (NDS 4.3.9) and the K_f factor for built-up columns (NDS 15.3.2). These factors were developed from mathematical modeling and testing, and as such, only apply to similarly constructed assemblies. Similar "system effect" adjustments can be found for wood studs (Section 2306.2.1 of 2006 IBC) and truss chord members (Section 6.4.2 of ANSI/TPI 1-2007). Lacking, however, is an explicit treatment of how to capitalize on the lower variability of effective modulus of elasticity for built-up members with respect to buckling calculations compared to buckling of individual members.

Current design practice for beam and column buckling capacities is to use the minimum modulus of elasticity value (E_{min}) that is tabulated for *individual* pieces of lumber. The shortcoming with this approach is that it ignores the averaging effect on stiffness that occurs when the mechanically fastened laminations are constrained to deflect to a common shape. Kimble and Bender (2010) accounted for reduced variability in *E* when considering stability calculations for mechanically built-up beams and columns. Their method capitalizes on the statistical averaging of E-values that occurs with built-up beams and columns constrained to deflect equally, and it is conservatively based on the assumption of no composite action between the plies of the built-up member. The objective of this paper is to present the buckling calculation method to post-frame building designers, along with examples to illustrate the method for a nail-laminated column and built-up header.

Current Characterization of E for Buckling Stress Calculations

The only material property used in the National Design Specification for Wood Construction (NDS) to calculate critical buckling stresses of timber beams and columns is the modulus of elasticity (E) (shear modulus for beam buckling is accounted for in terms of E). Tabulated values of E in the NDS represent *average* single-member properties for a species combination and grade. For column buckling analyses, the NDS stipulates that E_{min} should be used and it represents a lower 5% exclusion limit together with a safety factor of 1.66. Equation 1 shows the calculation of E_{min} as described in Appendix D of the NDS. Tabulated design values for E_{min} are based on assumed coefficients of variation of E (COV_E) for visually graded, machine evaluated, and machine-stress rated lumber grades of 25, 15 and 11 percent, respectively (NDS Appendix F).

$$E_{min} = \frac{1.03\,E\left[1 - 1.645\,COV_E\right]}{1.66} \quad (1)$$

Accounting for Reduced Variability in E Due to Averaging Effect

Woeste (1999) was the first to observe that when *n* members in a light-frame assembly are fastened together and constrained to bend in equal amounts, a mechanical "averaging" of the individual E values is achieved. Woeste applied this concept to explain why some wood truss roofs that lack the required permanent *diagonal* bracing to stabilize the Web continuous lateral restraints perform somewhat better than predicted by typical bracing design calculations.

For the case of built-up members that are constrained to deflect together, each of the *n* members in the built-up assembly has a distinct modulus of elasticity value (E_i) from the "population" of E-values for the grade. When the *n* members are constrained to bend (or buckle) to a common shape, mechanical "averaging" produces an *effective* system modulus of elasticity (E_{eff}). The effective E of a built-up beam or column with *n* members from the same population (or lumber grade) is simply the arithmetic average of the individual E_i values as follows

$$E_{eff} = E_1/n + E_2/n + \ldots + E_n/n \quad (2)$$

E_i and E_{eff} are both random variables with the same mean value, but E_{eff} has less variability due to the averaging effect. The coefficient of variation of E_{eff}, COV_{Eeff}, is given by

$$COV_{E\,eff} = \frac{COV_E}{\sqrt{n}} \quad (3)$$

The fifth percentile pure bending modulus of elasticity with a 1.66 factor of safety for a built-up beam or column with all plies constrained to deflect equally is given by

$$E_{eff-min} = \frac{1.03\,E\left[1 - 1.645\,COV_E/\sqrt{n}\right]}{1.66} \quad (4)$$

Any multi-ply beam or column with all individual members constrained to deflect equally through the application of proper design, detail and construction are hereafter referred to as E-averaged systems. The built-up header in Figure 1, for example, is an "E-averaged system" because the placement of the truss on top of the header, together with nailing along the header, forces the header plies to deflect to a common shape.

Kimble and Bender (2010) proposed Cs as an adjustment factor for E_{min} in stability calculations for mechanically built-up beams and columns installed as an E-averaged system. Cs is the ratio of Equation 4 to Equation 1 as follows

$$C_s = \frac{E_{eff-min}}{E_{min}} = \frac{\left[1 - 1.645\,COV_E/\sqrt{n}\right]}{\left[1 - 1.645\,COV_E\right]} \quad (5)$$

This adjustment capitalizes on the fact that built-up assemblies have less E variability than that of the individual constituent lumber. For example, a 4-ply nail-laminated column made from visually graded lumber would have an effective coefficient of variation of E equal to

$$COV_{Eeff} = \frac{25\%}{\sqrt{4}} = 12.5\% \quad (6)$$

This reduction in COV from 25% to 12.5% results in higher critical buckling stresses, and it conservatively ignores any partial composite action with respect to strength checks. Values of C_s based on lumber grading system and number of laminations are summarized in **Table 1**. As expected, the influence of the averaging effect shown in Table 1 is greatest for visually graded lumber since it has the highest COV_E and least highest for machine stress-rated lumber, which has the lowest COV_E.

Design Examples

Our design examples will follow the allowable stress design method in the 2005 NDS.

Nail-laminated column with strong axis buckling potential

Consider a 3-ply, nail-laminated column that is constructed using 2x8 No.1 Dense Southern Pine dimension lumber. The column is assumed to be embedded 4-feet with the bottom portion being preservative pressure-treated (PPT) lumber as shown in **Figure 1**. Wall girts are inset between columns at 36-in. on-center spacing, providing bracing against weak axis column buckling. The unbraced column length for strong axis buckling is 16 ft. Column buckling capacities with and without the C_s adjustment are presented to show the beneficial impact of the proposed method. Due to space limitations, the complete column design checks (including combined compression and bending) are not shown.

When designing columns, all applicable load combinations should be checked according to Chapter 2 of ASCE 7-05 (ASCE, 2005). Column designs in post-frame buildings are typically controlled by a combination of wind, snow and dead loads. For this case of combined bending and compression, the design criterion given in NDS equation 3.9-3 must be satisfied, as shown below. To illustrate the proposed E_{min} adjustment, we will limit our example to calculation of the allowable compression design stress, F'_c, for the case of strong axis buckling and only for the positive moment region of the column above the splice joint. All plies of the column are assumed to deflect in a common shape due to the nail laminating. The proposed adjustment to E_{min} can influence the terms F'_c, F'_b and F_{cE} in the following interaction equation.

$$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_b}{F'_b\left[1 - (f_c/F_{cE})\right]} \le 1.0$$

where:

$$f_c \le F_{cE} = \frac{0.822\,E'_{min}}{\left(L_e/d\right)^2}$$

and:
d = depth of column (strong axis)
E'_{min} = adjusted modulus of elasticity
f_b = actual strong axis bending stress
F'_b = adjusted allowable bending design stress
f_c = actual compression stress
F'_c = adjusted allowable compression parallel-to-grain design stress
F_{cE} = critical buckling design stress for compression members
L_e = effective column length (for strong axis buckling)

Solution

From Table 4B of the NDS Supplement, we obtain the tabulated design values and adjustment factors for 2x8 No. 1 Dense Southern Pine dimension lumber

F_c = 1,800 psi
C_D = 1.60 (load duration adjustment for wind, snow, plus dead load combinations)

The reference compression design value multiplied by all applicable factors except C_p is
$F_c^* = F_c * C_D$ = 1,800 psi * 1.6 = 2,880 psi

The tabulated single-member value of E_{min} is
E_{min} = 660,000 psi

For strong axis buckling, the three plies would be constrained to a common deflected shape because all plies are assumed to be connected full length by nail-laminating, so the proposed adjustment for E_{min} would apply. From Table 1 of this paper, we obtain the adjustment factor for E_{min} for the case of three plies of VSR lumber. The result is $C_s = 1.30$.

The final adjusted value of E_{min} is given by

$E'_{min} = E_{min} * C_s = 660{,}000 * 1.30 = 858{,}000$ psi

The effective buckling length, L_e, is calculated using NDS Appendix G, assuming fixed-pinned end conditions due to the 4-ft embedment.

$L_e = K_e * L = 0.8 * 192$ in $= 154$ in

The slenderness ratio is given by

$L/d = 154/7.25 = 21.2 < 50$ – checks OK as per NDS section 3.7.1.4

$c = 0.8$ for sawn lumber

The critical buckling stress design value is given by

$$F_{cE} = \frac{0.822\,E'_{min}}{(L_e/d)^2} = \frac{0.822 * 858{,}000}{(21.2)^2} = 1{,}570 \text{ psi}$$

Solving for the column stability factor using NDS equation 3.7-1

$$C_P = \frac{1+(F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1+(F_{cE}/F_c^*)}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}} = 0.465$$

The final adjusted allowable compression stress is given by

$F_c' = C_p * F_c^* = 0.465 * 2{,}880\text{psi} = 1{,}339$ psi

If C_s were not used to adjust E_{min}, a value of C_p equal to 0.374 would have resulted. *Thus, the proposed method yielded a 24% increase in allowable column buckling capacity for this example.*

Built-Up Header with Lateral Torsional Buckling Potential

Consider a nail-laminated header supporting a truss at mid-span as shown in Figure 1. The truss span is 48 ft with 20-4-0-1 psf loading (TC snow – TC dead – BC live – BC dead), assuming 29-gauge corrugated steel on 2x4 roof purlins and no ceiling. Trusses are spaced 8 ft on-center. The resulting truss reactions are 25 psf x 48/2 ft x 8 ft = 4,800 lbs. The header is assumed to be laterally restrained at the support columns and at the mid-span by the truss having diaphragm restraint due to the nailed or screwed steel roof sheathing. The distance between points of lateral support for the beam/header is therefore 8 ft.

Our proposed solution is a 4-ply 2x12 built-up header (nailed) using Dense Select Structural (DSS) Southern Pine dimension lumber. The design criteria include checks for bending, shear, deflection and bearing. Because of space limitations, only the bending check is shown in this article.

Solution

From **Table 4B** of the NDS Supplement, we obtained the tabulated design values and adjustment factors for 2x12 DSS Southern Pine lumber

$F_b = 2{,}050$ psi

$C_D = 1.15$ (load duration adjustment for snow plus dead load)

$C_r = 1.15$ (repetitive member increase to account for system effects on strength)

$E_{min} = 690{,}000$ psi

The reference bending design value multiplied by all applicable factors except C_L is

$F_b^* = F_b * C_D * C_r = 2{,}050 \text{ psi} * 1.15 * 1.15 = 2{,}711$ psi

In this case, the four plies would deflect together because the truss bears on the top of the header, so our proposed adjustment for E_{min} would apply. We obtain our adjustment factor from Table 1.

$C_s = 1.35$

The final adjusted value of E_{min} is given by

$E'_{min} = E_{min} * C_s = 690{,}000 \text{ psi} * 1.35 = 931{,}500$ psi

A note on system effects is needed here. Repetitive member factor, C_r, is a strength adjustment that only applies when 3 or more members are connected (see NDS 4.3.9). However, the proposed C_s factor for E-averaging applies to assemblies with two or more members. So, for a 2-ply header, the C_r factor would ***not apply*** *but the C_s factor would apply, provided the plies were constrained to deflect equally.*

Because the bearing conditions at the supports and load points are assumed to be adequate, the design criterion for this problem is $f_b \leq F'_b$.

In practice, additional checks (not involving the use of C_s) would be required for shear stress, deflection and compression perpendicular to grain and under the truss, but we only demonstrated the check for bending stress to illustrate the use of the adjustment factor C_s.

Maximum moment for a simply supported beam with center point load is given by $M = PL/4 = (4{,}800 \text{ lb} * 192 \text{ in}) / 4 = 230{,}400$ in-lb

The actual bending stress is the moment divided by four times the section modulus of one 2x12.

$f_b = M/S = 230{,}400 \text{ in-lb} / [4 * 31.64] = 1{,}820$ psi

The distance between points of lateral support, L_u, is 96 in (NDS section 3.3.3.4). The effective beam bending length (L_e) equation is determined from NDS Table 3.3.3 as follows

$L_e = 1.11\ L_u = 1.11 * 96 \text{ in} = 106.6$ in

The beam stability factor, C_L, is calculated using NDS Table 3.3.3 and Equations 3.3-5 and 3.3-6. *One issue requiring judgment is the choice of "b" in the slenderness ratio R_B. When multi-ply members comprised of 2-inch nominal thickness dimension lumber are nailed together, complete composite action is not achieved.* Hence, the torsional rigidity of the assembly is less than a solid member of the same size and it is not appropriate to use "b" as the total width of the plies. In this example, we make the conservative assumption of using the thickness of one ply (b = 1.5 in).

$$R_B = \sqrt{\frac{L_e\,d}{b^2}} = \sqrt{\frac{106.6\text{in} * 11.25\text{in}}{(1.5\text{in})^2}} = 23.1 < 50$$

Checks OK as per NDS section 3.3.3.7

$$F_{bE} = \frac{1.20\,E'_{min}}{R_B^{\,2}} = \frac{1.20 * 931{,}500\text{psi}}{(23.1)^2} = 2{,}095 \text{ psi}$$

$$C_L = \frac{1+(F_{bE}/F_b^*)}{1.9} - \sqrt{\left[\frac{1+(F_{bE}/F_b^*)}{1.9}\right]^2 - \frac{F_{bE}/F_b^*}{0.95}} = 0.694$$

The adjusted allowable bending design stress is given by

$F'_b = C_L\ F_b^* = 0.694 * 2{,}711 \text{ psi} = 1{,}881$ psi

Finally, we compare the applied bending stress to the adjusted allowable bending stress.

$f_b = 1{,}820 \text{ psi} < F'_b = 1{,}881$ psi.

Design checks OK in bending.

If C_s was not used to adjust E_{min}, the value of C_L would equal to 0.541. Thus, *the proposed method yielded a 28% increase in allowable bending stress.*

Table. 1. *C_s values based on n laminations and lumber grading method, and the assumption that all n-laminations are constrained to deflect equally by a design, detail, and installation that yields a common deflected shape.*

Number of laminations, n	Visual stress rated ($COV_E = 25\%$)	Machine evaluated lumber ($COV_E = 15\%$)	Machine stress rated lumber ($COV_E = 11\%$)
2	1.20	1.10	1.06
3	1.30	1.14	1.09
4	1.35	1.16	1.11
5	1.39	1.18	1.12

Without the proposed adjustment, the header design would not have checked for bending stress. It is important to note that the value of "b" used in the slenderness ratio was for one ply, since full composite action is not achieved in a mechanically built-up beam. When using the conservative assumption for "b" in NDS 3.3.3.7, the proposed adjustment to E_{min} for an "E-averaged system" is not only appropriate but it also produces a conservative bending strength design.

As a cautionary note, the adjustment factors in Table 1 should not be applied to the case of trusses or other framing side-mounted to a multi-ply header because the plies of the header would not have a common deflected shape or precisely equal deflections. Because of "slip" of the nail or bolt connections joining the side-loaded plies, the ply closest to the truss (or trusses) experiences more deflection than the second ply, and the second ply experiences more deflection than the 3rd ply, and so on. When using Table 1 for any design, the design professional should ensure through detailing

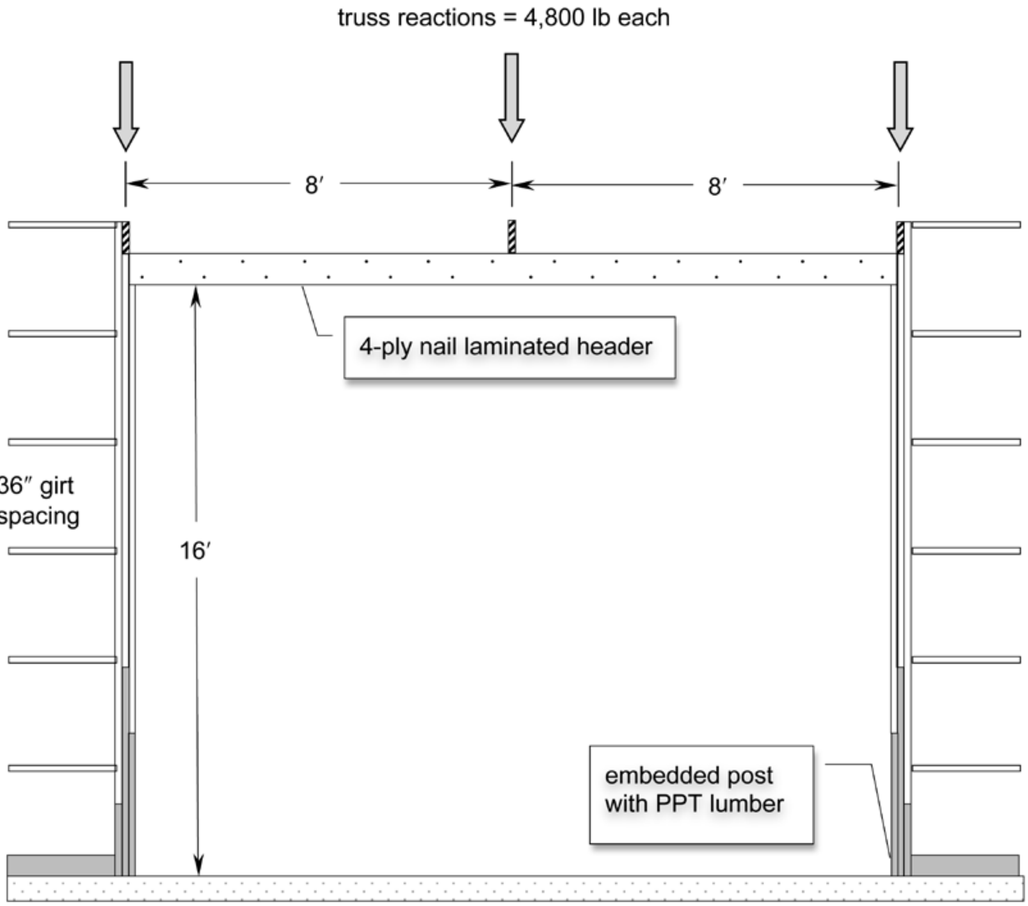

Figure 1. Example problem showing three-ply nail-laminated columns supporting a four-ply header.

that the individual plies are forced to deflect the same amount, which is a reasonable assumption for the example case presented herein (Figure 1).

Summary and Conclusions

Built-up beams and columns assembled from dimension lumber and fastened with nails, screws, or bolts exhibit less variability in modulus of elasticity as compared to individual pieces of the constituent lumber. This reduced variability is the result of "E-averaging" that occurs when *n* members are constrained to deflect into a common shape. Allowable stress adjustment factors are available in the NDS and other sources (e.g., TPI and IBC) to account for load sharing "system effects" with respect to strength checks. Lacking to date has been an explicit method to capitalize on the reduced variability in E for determining the buckling capacity of built-up beams and columns.

Buckling capacity calculations for built-up beams and columns utilize the property E_{min}, which represents a lower-tail value of modulus of elasticity with a safety factor of 1.66. The NDS Supplement tabulates *single-member* values of E_{min} for dimension lumber. In this paper, we present an adjustment factor for E_{min} to account for the reduction in variability of E when multiple plies of dimension lumber are constrained to deflect equally by proper design, detail and installation producing what is defined in this article as an E-averaged system. Two examples were given to illustrate the method – a built-up side wall column and 16-ft. door header. The benefits of an E-averaged system depend on the specifics of the design application such as number of plies, member depth, unsupported length of a built-up beam, column length, and so on. Based on the adjustment factors in Table 1, it is clear that VSR lumber accrues the greatest benefit of the design approach that relies on an E-averaged system installation.

References

American Forest and Paper Association. (2005). *ANSI/AF&PA NDS-2005 National Design Specification for Wood Construction ASD/LRFD.* Retrieved April 19, 2010, from http://www.awc.org

American Society of Civil Engineers (ASCE). (2005). *ASCE/SEI 7-05 Minimum Design Loads for Buildings and Other Structures.* Retrieved April 19, 2010, from http://www.asce.org

International Code Council (ICC). (2006). *International Building Code.* Retrieved April 19, 2010, from http://www.iccsafe.org.

Kimble, R. E. & Bender, D. A. (2010). Stability of built-up timber beams and columns — accounting for modulus of elasticity variability. *ASCE Practice Periodical on Structural Design and Construction (in print).*

Truss Plate Institute (TPI). (2007). *ANSI/TPI 1-2007 National Design Standard for Metal Plate Connected Wood Truss Construction.* Retrieved April 19, 2010, from www.tpinst.org

Woeste, F. E. (1999). Impact of permanent diagonal bracing on different MPC wood truss types. *ASCE Practice Periodical on Structural Design and Construction, 4(1), 21-23.* FBN